Example 8.6

Example (8.1.6) is solved in Maple and the results obtained are given below:

```
> restart : with(inttrans) : with(plots) :
> eq:=diff(u(x,t),t)=diff(u(x,t),x$2)+2/x*diff(u(x,t),x);
```

$$eq := \frac{\partial}{\partial t} u(x,t) = \frac{\partial^2}{\partial x^2} u(x,t) + \frac{2\left(\frac{\partial}{\partial x} u(x,t)\right)}{x} \tag{1}$$

```
> u(x,0):=0;
```

$$u(x,0) := 0 \tag{2}$$

```
> bc1:=diff(u(x,t),x)=0;
```

$$bc1 := \frac{\partial}{\partial x} u(x,t) = 0 \tag{3}$$

```
> bc2:=u(x,t)=1;
```

$$bc2 := u(x,t) = 1 \tag{4}$$

```
> eqs:=laplace(eq,t,s):
> eqs:=subs(laplace(u(x,t),t,s)=U(x),eqs);
```

$$eqs := s\,U(x) = \frac{d^2}{dx^2} U(x) + \frac{2\left(\frac{d}{dx} U(x)\right)}{x} \tag{5}$$

```
> bc1:=laplace(bc1,t,s):
> bc1:=subs(diff(laplace(u(x,t),t,s),x)=D(U)(0),laplace(u(x,t),t,
  s)=U(0),bc1);
```

$$bc1 := D(U)(0) = 0 \tag{6}$$

```
> bc2:=laplace(bc2,t,s):
> bc2:=subs(diff(laplace(u(x,t),t,s),x)=D(U)(1),laplace(u(x,t),t,
  s)=U(1),bc2);
```

$$bc2 := U(1) = \frac{1}{s} \tag{7}$$

```
> U(x):=rhs(dsolve({eqs,bc2},U(x)));
```

$$U(x) := \frac{\_C1 \sinh\left(\sqrt{s}\,x\right)}{x} - \frac{\left(s\,\_C1\,e^{8\sqrt{s}} - s\,\_C1\,e^{6\sqrt{s}} - 2\,e^{7\sqrt{s}}\right)\cosh\left(\sqrt{s}\,x\right)}{s\left(e^{8\sqrt{s}} + e^{6\sqrt{s}}\right)x} \tag{8}$$

```
> U(x):=subs(_C2=0,U(x));
```

$$U(x) := \frac{\_C1 \sinh\left(\sqrt{s}\,x\right)}{x} - \frac{\left(s\,\_C1\,e^{8\sqrt{s}} - s\,\_C1\,e^{6\sqrt{s}} - 2\,e^{7\sqrt{s}}\right)\cosh\left(\sqrt{s}\,x\right)}{s\left(e^{8\sqrt{s}} + e^{6\sqrt{s}}\right)x} \tag{9}$$

```
> convert(U(x),exp);
```

$$\frac{\_C1\left(\frac{1}{2}\,e^{\sqrt{s}\,x} - \frac{1}{2}\,e^{-\sqrt{s}\,x}\right)}{x} \tag{10}$$

$$-\frac{\left(s\_C1\, e^{8\sqrt{s}} - s\_C1\, e^{6\sqrt{s}} - 2\, e^{7\sqrt{s}}\right)\left(\frac{1}{2}\, e^{\sqrt{s}\, x} + \frac{1}{2}\, e^{-\sqrt{s}\, x}\right)}{s\left(e^{8\sqrt{s}} + e^{6\sqrt{s}}\right) x}$$

```
> U1s:=exp(s^(1/2))/s/(exp(s^(1/2))^2-1)*exp(s^(1/2)*x)/x;
```

$$U1s := \frac{e^{\sqrt{s}}\, e^{\sqrt{s}\, x}}{s\left(\left(e^{\sqrt{s}}\right)^2 - 1\right) x} \tag{11}$$

```
> U2s:=-exp(s^(1/2))/s/(exp(s^(1/2))^2-1)*exp(-s^(1/2)*x)/x;
```

$$U2s := -\frac{e^{\sqrt{s}}\, e^{-\sqrt{s}\, x}}{s\left(\left(e^{\sqrt{s}}\right)^2 - 1\right) x} \tag{12}$$

```
> U1S:=series(subs(exp(s^(1/2))=1/S,U1s),S):
> U1S:=subs(S=exp(-s^(1/2)),U1S):
> simplify(U1S);
```

$$\frac{e^{\sqrt{s}\,(x-1)} + e^{\sqrt{s}\,(x-3)} + e^{\sqrt{s}\,(x-5)} + O\left(e^{-7\sqrt{s}}\right) s\, x}{s\, x} \tag{13}$$

```
> U1S:=1/x*Sum(exp(s^(1/2)*(x-2*n+1))/s,n=1..infinity);
```

$$U1S := \frac{\sum_{n=1}^{\infty} \frac{e^{\sqrt{s}\,(x-2n+1)}}{s}}{x} \tag{14}$$

```
> u1s:=exp(s^(1/2)*(x-2*n+1))/s;
```

$$u1s := \frac{e^{\sqrt{s}\,(x-2n+1)}}{s} \tag{15}$$

```
> u1t:=invlaplace(u1s,s,t);
```

$$u1t := invlaplace\left(\frac{e^{\sqrt{s}\,(x-2n+1)}}{s}, s, t\right) \tag{16}$$

```
> U1t:=1/x*Sum(u1t,n=1..infinity);
```

$$U1t := \frac{\sum_{n=1}^{\infty} invlaplace\left(\frac{e^{\sqrt{s}\,(x-2n+1)}}{s}, s, t\right)}{x} \tag{17}$$

```
> U2S:=series(subs(exp(s^(1/2))=1/S,U2s),S):
> U2S:=subs(S=exp(-s^(1/2)),U2S):
> simplify(U2S);
```

$$-\frac{e^{-\sqrt{s}\,(x+1)} + e^{-\sqrt{s}\,(x+3)} + e^{-\sqrt{s}\,(x+5)} - O\left(e^{-7\sqrt{s}}\right) s\, x}{s\, x} \tag{18}$$

```
> U2S:=-1/x*Sum(exp(-s^(1/2)*(x+2*n-1))/s,n=1..infinity);
```

$$U2S := -\frac{\sum_{n=1}^{\infty} \frac{e^{-\sqrt{s}\,(x+2n-1)}}{s}}{x} \tag{19}$$

```
> u2s:=exp(-s^(1/2)*(x+2*n-1))/s:
> u2t:=invlaplace(u2s,s,t);
```

$$u2t := invlaplace\left(\frac{e^{(-x-2n+1)\sqrt{s}}}{s}, s, t\right) \tag{20}$$

```
> U2t:=-1/x*Sum(u2t,n=1..infinity);
```

$$U2t := -\frac{\sum_{n=1}^{\infty} invlaplace\left(\frac{e^{(-x-2n+1)\sqrt{s}}}{s}, s, t\right)}{x} \tag{21}$$

```
> Ut:=U1t+U2t;
```

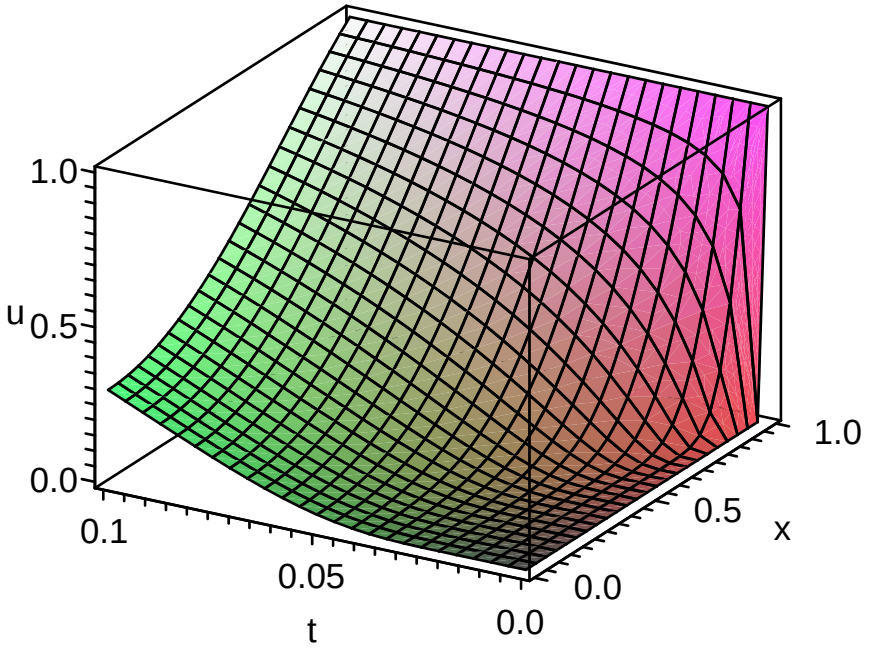
$$Ut := \frac{\sum_{n=1}^{\infty} invlaplace\left(\frac{e^{\sqrt{s}\,(x-2n+1)}}{s}, s, t\right)}{x} - \frac{\sum_{n=1}^{\infty} invlaplace\left(\frac{e^{(-x-2n+1)\sqrt{s}}}{s}, s, t\right)}{x} \tag{22}$$

```
> u:=subs(infinity=N,Ut):
> u:=subs(N=20,u);
```

$$u := \frac{\sum_{n=1}^{20} invlaplace\left(\frac{e^{\sqrt{s}\,(x-2n+1)}}{s}, s, t\right)}{x} - \frac{\sum_{n=1}^{20} invlaplace\left(\frac{e^{(-x-2n+1)\sqrt{s}}}{s}, s, t\right)}{x} \tag{23}$$

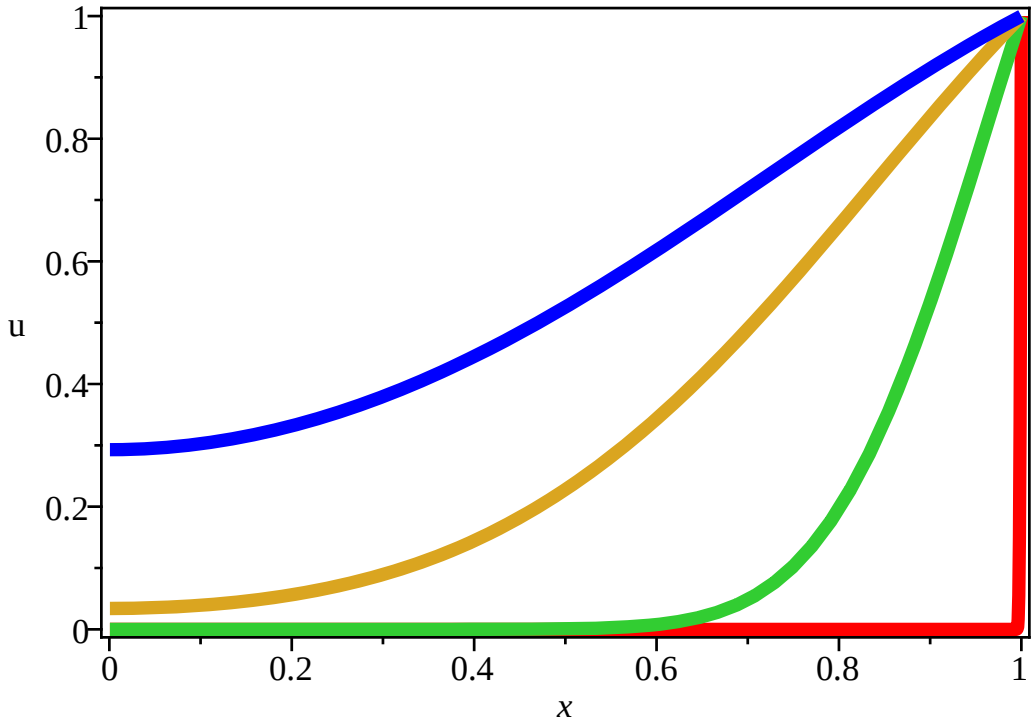
```
> plot3d(u,x=1e-6..1,t=1e-6..0.1,axes=boxed,title="Figure Exp.
  8.11.",labels=[x,t,"u"],orientation=[-150,60]);
```

Figure Exp. 8.11.




```
> plot([subs(t=1e-6,u),subs(t=1e-2,u),subs(t=0.05,u),subs(t=0.1,u)
  ],x=0..1,axes=boxed,title="Figure Exp. 8.12.",thickness=5,
  labels=[x,"u"]);
```

Figure Exp. 8.12.



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